Killing reduction of 5-dimensional spacetimes

Xuejun Yang,* Yongge Ma,† Jianbing Shao, and Wei Zhou
Department of Physics, Beijing Normal University, Beijing 100875, China
(Received 9 April 2003; published 3 July 2003)

In a 5-dimensional spacetime (M,g_{ab}) with a Killing vector field ξ^a which is either everywhere time like or everywhere space like, the collection of all trajectories of ξ^a gives a 4-dimensional space S. The reduction of (M,g_{ab}) is studied in the geometric language, which is a generalization of Geroch's method for the reduction of 4-dimensional spacetime. A 4-dimensional gravity coupled to a vector field and a scalar field on S is obtained by the reduction of vacuum Einstein's equations on M, which gives also an alternative description of the 5-dimensional Kaluza-Klein theory. In addition to the symmetry-reduced action from the Hilbert action on M, an alternative action of the fields on S is also obtained, the variations of which lead to the same fields equations as those reduced from the vacuum Einstein equation on M.

DOI: 10.1103/PhysRevD.68.024006 PACS number(s): 04.50.+h, 04.20.Fy

I. INTRODUCTION

Spacetime reduction is very important in any high dimensional theory of physics such as Kaluza-Klein (KK) theory [1–5], high dimensional theory of gravity [6,7], and string theory [8,9]. Dimensional reduction can make a high dimensional theory contact the 4-dimensional sensational world. It is also a useful approach to study spacetimes with symmetries. The original Kaluza-Klein theory unifies electromagnetic and gravitational interactions in four dimensions by a 5-dimensional spacetime. Since the theory was first proposed by Kaluza [10], it has been studied by a large number of authors [1–5] and also extended to higher dimensions in order to give rise to a unification of gravity with non-Abelian gauge theories [11]. Along with the successful description of the electroweak and strong interactions as gauge theories, the KK approach may serve as a framework for studying their unification with gravity. Also using the idea of supergravity in KK theory, a geometric description of both gauge fields and spinor matter is possible.

In Ref. [12], Geroch introduced a Killing reduction formalism of 4-dimensional spacetime. Let $(\mathcal{M},g_{ab})$ be a 4-dimensional spacetime with a Killing vector field ξ^a which is either everywhere time like or everywhere space like. The collection of all trajectories of ξ^a gives a 3-dimensional space Σ. The discussion of Geroch shows that there is a one-to-one correspondence between tensor fields and tensor operations on Σ and the certain tensor fields and tensor operations on $\mathcal{M}$. The differential geometry of Σ will, in this sense, be mirrored in $\mathcal{M}$. Geroch gives the relations of geometric properties between Σ and $\mathcal{M}$ and then obtains the field equations on Σ, which describes a 3-dimensional gravity coupled to two massless scalar fields and is equivalent to the vacuum Einstein equations on $\mathcal{M}$.

In this paper we extend Geroch's approach to a 5-dimensional spacetime (M,g_{ab}) with a Killing vector field in order to study its reduction. We first obtain a series of equations on S, which is also the collection of all trajectories of ξ^a, parallel to Geroch's. The results show that 5-dimensional gravity with a Killing vector is equivalent to 4-dimensional gravity h_{ab} coupled to a vector field A_a and a scalar field λ, and hence it is consistent with the conclusion of the Kaluza-Klein theory. We then study the reduction from the viewpoint of variation principle. It turns out that the symmetry-reduced action from the Hilbert action on M would give the correct reduced field equations only if one used B_a, which has a trivial relation with the 5-metric components, rather than A_a as one of the arguments. Finally, we propose another 4-dimensional action on S. Its variations with respect to h^{ab}, λ and A_a can give the same reduced field equations.

II. SYMMETRIC REDUCTION OF 5-DIMENSIONAL SPACETIME

Let (M,g_{ab}) be an n-dimensional spacetime with a Killing vector field ξ^a, which is either everywhere time like or everywhere space like. Let S denote the collection of all trajectories of ξ^a. A map ψ from M to S can be defined as follows: For each point p of M, $\psi(p)$ is the trajectory of ξ^a passing through p. Assume S is given the structure of a differentiable $(n-1)$-manifold such that ψ is a smooth mapping. It is natural to regard S as a quotient space of M. The proof of Geroch about the following conclusion is independent of the dimension of M: There is a one-to-one correspondence between tensor fields $\tilde{T}^{b\cdots d}_{a\cdots c}$ on S and tensor fields $T^{b\cdots d}_{a\cdots c}$ on M which satisfy

$$\xi^a T^{b\cdots d}_{a\cdots c}=0,\ldots,\xi_d T^{b\cdots d}_{a\cdots c}=0,$$
$$\mathcal{L}_\xi T^{b\cdots d}_{a\cdots c}=0. \tag{1}$$

The entire tensor field algebra on S is completely and uniquely mirrored by tensor field on M subject to Eqs. (1). Thus, we shall speak of tensor fields being on S merely as a shorthand way of saying that the fields on M satisfy Eqs. (1).

The metric, inverse metric and the Kronecker delta on S are defined as

$$h_{ab}=g_{ab}-\lambda^{-1}\xi_a\xi_b, \tag{2}$$

*Email address: yang_xue_jun@sohu.com
†Email address: yonggema@yahoo.com

$$h^{ab}=g^{ab}-\lambda^{-1}\xi^a\xi^b, \tag{3}$$

$$h_a^b=\delta_a^b-\lambda^{-1}\xi_a\xi^b, \tag{4}$$

where $\lambda\equiv\xi^a\xi_a$. Equation (4) can also be interpreted as the projection operator onto S. The covariant derivative on S is defined by

$$D_eT_{a\cdots c}^{b\cdots d}=h_e^ph_a^m\cdots h_c^nh_r^b\cdots h_s^d\nabla_pT_{m\cdots n}^{r\cdots s}, \tag{5}$$

where ∇_p is the covariant derivative associated with the metric g_{ab} on M and $T_{a\cdots c}^{b\cdots d}$ is any tensor field on S. Note that D_e satisfies all the conditions of a derivative operator and $D_ch_{ab}=0$.

We now consider the special case where $n=5$. It can be shown that $\varepsilon_{abcd}\equiv|\lambda|^{-1/2}\varepsilon_{abcde}\xi^e$ is the volume element associated with the metric h_{ab} on S, i.e.,

$$D_f\varepsilon_{abcd}=0; \tag{6}$$

here, ε_{abcde} is the volume element associated with the metric g_{ab} on M, i.e., $\nabla_f\varepsilon_{abcde}=0$. We define the twist 2-form ω_{ab} of ξ^a by

$$\omega_{ab}:=\varepsilon_{abcde}\xi^c\nabla^d\xi^e. \tag{7}$$

Clearly we have $\omega_{ab}=\omega_{[ab]}$ and $\omega_{ab}\xi^a=0$. It is also easy to see that

$$\mathcal{L}_\xi\lambda=0,\quad \mathcal{L}_\xi\omega_{ab}=0. \tag{8}$$

Hence, λ and ω_{ab} are fields on S.

Lengthy but straightforward calculations lead to the following results.

(1) The Riemann tensor R_{abcd} of (S,h_{ab}) is related to the Riemann tensor $\mathcal{R}_{abcd}$ of (M,g_{ab}) by

$$R_{abcd}=h_{[a}^ph_{b]}^qh_{[c}^rh_{d]}^s[\mathcal{R}_{pqrs}+2\lambda^{-1}(\nabla_p\xi_q)(\nabla_r\xi_s)+2\lambda^{-1}(\nabla_p\xi_r)(\nabla_q\xi_s)]. \tag{9}$$

(2) The derivative of the Killing vector reads

$$\nabla_a\xi_b=-\frac{1}{4}\lambda^{-1}\varepsilon_{abcde}\xi^c\omega^{de}+\lambda^{-1}\xi_{[b}D_{a]}\lambda. \tag{10}$$

(3) The second derivative reads

$$\nabla_a\nabla_b\xi_c=\mathcal{R}_{dabc}\xi^d. \tag{11}$$

(4) Contracting Eq. (9) and using Eqs. (10) and (11), the relation between the Ricci tensor R_{ab} of (S,h_{ab}) and the Ricci tensor $\mathcal{R}_{ab}$ of (M,g_{ab}) can be obtained as

$$R_{ab}=\frac{1}{2}\lambda^{-2}\left(\omega_a^m\omega_{bm}-\frac{1}{2}h_{ab}\omega_{mn}\omega^{mn}\right)+\frac{1}{2}\lambda^{-1}D_aD_b\lambda-\frac{1}{4}\lambda^{-2}(D_a\lambda)D_b\lambda+h_a^mh_b^n\mathcal{R}_{mn}. \tag{12}$$

(5) Taking the curl and divergence of Eq. (7) and using Eqs. (10) and (11), one can get

$$D_{[a}\omega_{bc]}=\frac{2}{3}\varepsilon_{abcmn}\xi^m\mathcal{R}_p^{\ n}\xi^p \tag{13}$$

and

$$D^a\omega_{ab}=\frac{3}{2}\lambda^{-1}\omega_{mb}D^m\lambda. \tag{14}$$

(6) Using Eqs. (10) and (11) again, we have

$$D^2\lambda=\frac{1}{2}\lambda^{-1}(D^a\lambda)D_a\lambda-\frac{1}{2}\lambda^{-1}\omega^{ab}\omega_{ab}-2\mathcal{R}_{mn}\xi^m\xi^n, \tag{15}$$

where $D^2\equiv D^aD_a$.

(7) Contracting Eq. (12) by h^{ab} and using Eq. (15), we obtain

$$R=-\frac{1}{4}\lambda^{-2}\omega_{mn}\omega^{mn}+\lambda^{-1}D^2\lambda-\frac{1}{2}\lambda^{-2}(D^a\lambda)D_a\lambda+\mathcal{R}; \tag{16}$$

here, R and $\mathcal{R}$ are the scalar curvature of (S,h_{ab}) and (M,g_{ab}) respectively.

Thus, the basic equations for a 5-dimensional spacetime with a Killing vector field are a set of differential equations on three variables, the metric h^{ab}, the norm λ and the twist 2-form ω_{ab} of the Killing vector. They are formulated as Eqs. (12), (13), (14), and (15).

When the spacetime (M,g_{ab}) is source free $(\mathcal{R}_{ab}=0)$, Eq. (13) implies that $(d\omega)_{abc}$ on S is zero. Hence at least locally there is a 1-form A_a on S such that $\omega_{ab}=(dA)_{ab}\equiv 2D_{[a}A_{b]}$. Then Eqs. (12)–(15) become

$$R_{ab}=\frac{1}{2}\lambda^{-2}\left[(dA)_a^m(dA)_{bm}-\frac{1}{2}h_{ab}(dA)_{mn}(dA)^{mn}\right]+\frac{1}{2}\lambda^{-1}D_aD_b\lambda-\frac{1}{4}\lambda^{-2}(D_a\lambda)D_b\lambda, \tag{17}$$

$$D^a(dA)_{ab}=\frac{3}{2}\lambda^{-1}(dA)_{mb}D^m\lambda, \tag{18}$$

$$D^2\lambda=\frac{1}{2}\lambda^{-1}(D^a\lambda)D_a\lambda-\frac{1}{2}\lambda^{-1}(dA)_{ab}(dA)^{ab}. \tag{19}$$

Equations (17)–(19) describe a 4-dimensional gravity coupled to a vector field and a scalar field, which are equivalent to a 5-dimensional vacuum Einstein's equation with a Killing symmetry. When the Killing vector field is space like, it coincides with the conclusion of the 5-dimensional Kaluza-Klein theory. Thus, by reducing Einstein's equation on M by an extension of Geroch's method, we can provide an alternative description of the Kaluza-Klein theory in 4+1 dimensions.

III. SYMMETRY-REDUCED HILBERT ACTION ON *S*

For practical calculations, it is convenient to take a coordinate system adapted to the congruence:

$$\left(\frac{\partial}{\partial x^5}\right)^a = \xi^a. \tag{20}$$

From Eq. (2), we have

$$g_{ab} = h_{ab} + \lambda^{-1}\xi_a \xi_b . \tag{21}$$

The components of Eq. (21) are

$$g_{\mu\nu} = h_{\mu\nu} + \lambda^{-1}\xi_\mu \xi_\nu, \quad \mu,\nu = 1,\dots,5. \tag{22}$$

Particularly one has

$$g_{\mu 5} = g_{ab}\left(\frac{\partial}{\partial x^\mu}\right)^a \xi^b = \left(\frac{\partial}{\partial x^\mu}\right)^a \xi_a = \xi_\mu \tag{23}$$

and

$$g_{55} = g_{ab}\xi^a \xi^b = \left(\frac{\partial}{\partial x^5}\right)^a \xi_a = \xi_5 = \lambda. \tag{24}$$

Let

$$B'_a = \lambda^{-1}\xi_a - (dx^5)_a ; \tag{25}$$

then, $\xi^a B'_a = 0$. One can also prove that $\mathcal{L}_\xi B'_a = 0$; hence, B'_a is a 1-form on S. Note that B'_a is dependent on the coordinate system chosen. Using Eqs. (10) and (25), it can be shown that

$$(dB')_{ba} \equiv 2D_{[b}B'_{a]} = \frac{1}{2}|\lambda|^{-3/2}\varepsilon_{abcd}\omega^{cd} \equiv \overline{\omega}_{ba} . \tag{26}$$

Equation (26) means $(d\overline{\omega})_{cba} = 0$; hence, there is at least locally a 1-form B_a on S such that $\overline{\omega}_{ba} = (dB)_{ba}$. This definition of B_a is independent of any coordinate system and hence purely geometric. Thus, we obtain

$$(dB)_{ba} = \frac{1}{2}|\lambda|^{-3/2}\varepsilon_{abcd}\omega^{cd},$$

$$\omega_{ab} = \pm\frac{1}{2}|\lambda|^{3/2}\varepsilon_{abcd}(dB)^{cd}. \tag{27}$$

Hereafter, when $\lambda > 0$ or $\lambda < 0$, the sign "$\pm$" means "$+$" or "$-$" respectively. The Hilbert action on M reads

$$S[g^{ab}] = \int_M \sqrt{-g}\,\mathcal{R}. \tag{28}$$

Since the principle of symmetric criticality is valid in our one Killing vector model [13], one expects that the reduced field, Eqs. (17)–(19), could be obtained by variation of the action from the symmetric reduction of Eq. (28). Using Eq. (22), we obtain

$$g = \lambda h, \tag{29}$$

where g and h are respectively the determinants of components $g_{\mu\nu}$ $(\mu,\nu = 1,2,\dots,5)$ and $h_{\mu\nu}$ $(\mu,\nu = 1,\dots,4)$. Using Eqs. (16) and (29), the action (28) on M is reduced to the following action on S up to a boundary term:

$$S[h^{ab},\lambda,\omega_{ab}] = \int_S |\lambda|^{1/2}\sqrt{|h|}\left[R + \frac{1}{4}\lambda^{-2} w_{mn} w^{mn}\right]. \tag{30}$$

The variation of action (30) with respect to $(h^{ab},\ \lambda,\ \omega_{mn})$ cannot give the correct reduced field equations. In order to get those equations, one has to use the 5-metric components rather than ω_{mn} as the arguments of the reduced action. This subtlety exists also in the symmetric reduction of 4-dimensional spacetimes [14].

From Eq. (27), we have

$$\omega^{mn}\omega_{mn} = -\lambda^3 (dB)_{ab}(dB)^{ab}. \tag{31}$$

Substituting Eq. (31) into Eq. (30), we obtain the action on S in terms of basic variables h^{ab}, λ, and B_a as

$$S[h^{ab},\lambda,B_a] = \int_S |\lambda|^{1/2}\sqrt{|h|}\left[R - \frac{1}{4}\lambda (dB)_{ab}(dB)^{ab}\right], \tag{32}$$

which has the same form as the reduced action in 5-dimensional KK theory. The variations of this action with respect to $(h^{ab},\ \lambda,\ B_c)$ will give the correct reduced field equations on S. Especially, the variation of action (32) with respect to B_a gives

$$D^c (dB)_{ac} = -\frac{3}{2}\lambda^{-1}(D^c\lambda)(dB)_{ac} . \tag{33}$$

Substituting Eq. (27) into Eq. (33), we get

$$D_{[a}\omega_{bc]} = 0. \tag{34}$$

Equation (34) means that there is at least locally a 1-form A_a on S such that

$$\omega_{ab} = (dA)_{ab} . \tag{35}$$

Substituting A_a for B_a in Eq. (33) through Eqs. (35) and (27), we obtain Eq. (18). Equations (17) and (19) can also be obtained by substituting A_a for B_a after the corresponding variations.

IV. ALTERNATIVE ACTION ON *S*

If one wants to take h^{ab}, λ, and A_a as basic variables, action (30) fails to be the right action. Another action is thus needed. When the Killing field ξ^a is hypersurface orthogonal, from Eqs. (23), (24), (25), (26), and (16), we have

$$\omega_{ab} = 0, \tag{36}$$

$$\mathcal{R} = R - \lambda^{-1} h^{ab} D_a D_b \lambda + \frac{1}{2}\lambda^{-2} h^{ab}(D_a\lambda) D_b \lambda. \tag{37}$$

In this case, up to a boundary term the Hilbert action (28) on M is reduced on S as [13]

$$S[h^{ab},\lambda]=\int_S |\lambda|^{1/2}\sqrt{|h|}R. \quad (38)$$

To obtain a regular form of action on S, we conformally transform h^{ab} as

$$\tilde{h}_{ab}=\Omega^{-2}h_{ab}. \quad (39)$$

Let $\tilde{D}_a$ be the covariant derivative operator determined by metric $\tilde{h}_{ab}$, i.e.,

$$\tilde{D}_a\tilde{h}_{bc}=0. \quad (40)$$

The relation of $\tilde{D}_a$ and D_a is [15]

$$D_a v_b=\tilde{D}_a v_b - C^c_{ab}v_c, \quad \forall \; v_a\in\mathcal{F}_S(0,1), \quad (41)$$

where

$$C^c_{ab}=\frac{1}{2}h^{cd}(\tilde{D}_a h_{bd}+\tilde{D}_b h_{ad}-\tilde{D}_d h_{ab})$$
$$=\tilde{h}^c_b\tilde{D}_a \ln\Omega+\tilde{h}^c_a\tilde{D}_b\ln\Omega-\tilde{h}_{ab}\tilde{h}^{cd}\tilde{D}_d\ln\Omega. \quad (42)$$

Let

$$\Omega=|\lambda|^{-1/4}. \quad (43)$$

Ignoring the boundary term Eq. (38) becomes

$$S[\tilde{h}^{ab},\lambda]=\int_S\sqrt{|\tilde{h}|}\left[\tilde{R}-\frac{3}{8}\lambda^{-2}\tilde{h}^{ab}(\tilde{D}_a\lambda)\tilde{D}_b\lambda\right]. \quad (44)$$

Let $\Lambda=\sqrt{6}\ln\Omega$; then, Eq. (44) becomes

$$S[\tilde{h}^{ab},\Lambda]=\int_S\sqrt{|\tilde{h}|}[\tilde{R}-\tilde{h}^{ab}(\tilde{D}_a\Lambda)\tilde{D}_b\Lambda]. \quad (45)$$

This is the action of a 4-gravity $\tilde{h}^{ab}$ coupled to a massless scalar field. Thus in the source-free case, a 5-dimensional spacetime with a hypersurface orthogonal Killing vector field which is either everywhere time like or everywhere space like is "conformally" equivalent to 4-dimensional gravity coupled to a massless Klein-Gordon field.

In general case, from Eqs. (17)–(19) we know that besides a scalar field λ, the 4-gravity couples also to a vector field A_a on S. By carefully observing the reduced field equations and the special action (44), we suppose the following action being the one we are looking for:

$$S[\tilde{h}^{ab},\lambda,A_a]=\int_S\sqrt{|\tilde{h}|}\left[\tilde{R}-\frac{3}{8}\lambda^{-2}\tilde{h}^{ab}(\tilde{D}_a\lambda)\tilde{D}_b\lambda\right.$$
$$\left.-\frac{1}{4}|\lambda|^{-3/2}\tilde{h}^{ac}\tilde{h}^{bd}(dA)_{ab}(dA)_{cd}\right]. \quad (46)$$

Varying the action (46) with respect to $\tilde{h}^{ab}$, λ, and A_a, respectively, we get

$$\tilde{R}_{ab}=\frac{3}{8}\lambda^{-2}(\tilde{D}_a\lambda)\tilde{D}_b\lambda-\frac{1}{8}|\lambda|^{-3/2}\tilde{h}_{ab}\tilde{h}^{ec}\tilde{h}^{fd}$$
$$\times(dA)_{ef}(dA)_{cd}+\frac{1}{2}|\lambda|^{-3/2}\tilde{h}^{cd}(dA)_{ac}(dA)_{bd}, \quad (47)$$

$$\tilde{h}^{ab}\tilde{D}_a\tilde{D}_b\lambda=\lambda^{-1}\tilde{h}^{ab}(\tilde{D}_a\lambda)\tilde{D}_b\lambda$$
$$+\frac{1}{2}|\lambda|^{-1/2}\tilde{h}^{ac}\tilde{h}^{bd}(dA)_{ab}(dA)_{cd}, \quad (48)$$

$$D^a(dA)_{ab}=\frac{3}{2}\lambda^{-1}(dA)_{ab}D^a\lambda. \quad (49)$$

Making the conformal transformation (39) inversely, Eqs. (47) and (48) become

$$R_{ab}=\frac{1}{2}\lambda^{-2}\left[h^{cd}(dA)_{ac}(dA)_{bd}-\frac{1}{2}h_{ab}h^{ec}h^{fd}(dA)_{ef}(dA)_{cd}\right]$$
$$+\frac{1}{2}\lambda^{-1}D_aD_b\lambda-\frac{1}{4}\lambda^{-2}(D_a\lambda)D_b\lambda, \quad (50)$$

$$D^2\lambda=\frac{1}{2}\lambda^{-1}h^{ab}(D_a\lambda)D_b\lambda-\frac{1}{2}\lambda^{-1}h^{ac}h^{bd}(dA)_{ab}(dA)_{cd}. \quad (51)$$

Equations (50), (49), and (51) are exactly the same as Eqs. (17)–(19). Therefore, the action (46) is just the one which gives also the right reduced field equations.

In conclusion, we have studied the symmetric reduction of 5-dimensional spacetime in three hands. First, we obtain 4-dimensional gravity coupled to a vector field and a scalar field on S by a direct reduction of the vacuum Einstein's equation on M. Then, taking h^{ab}, λ, and B_a as basic variables, we get reduced field equations by varying the symmetry-reduced action on S, which is obtained by the reduction of the Hilbert action on M. Finally, we propose an alternative action on S, which allows us to take h^{ab}, λ, and A_a as basic variables, and its variations gives also the right reduced field equations. All discussions are geometrically presented. The scheme might also be extended to higher-dimensional Kaluza-Klein theories that attempt to unify gauge theories with gravity.

ACKNOWLEDGMENTS

This work is supported in part by NSFC (grant 10205002), YSRF for ROCS, SEM, and Young Teachers Foundation of BNU. X.Y. would also like to acknowledge support from NSFC (grant 10073002).

[1] M.J. Duff and B.E.W. Nilsson, Phys. Rep. **130**, 1 (1986).
[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999).
[3] T. Han, J.D. Lykken, and R. Zhang, Phys. Rev. D **59**, 105006 (1999).
[4] G.F. Giudice, R. Rattazzi, and J.D. Wells, Nucl. Phys. **B544**, 3 (1999).
[5] F. Darabi and P.S. Wesson, Phys. Lett. B **527**, 1 (2002).
[6] J.M. Overduin and P.S. Wesson, Phys. Rep. **283**, 305 (1997).
[7] S. Weinberg, Phys. Lett. **125B**, 265 (1983).
[8] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vols. I and II.
[9] H.J. de Vega and N. Sanchez, Phys. Lett. B **197**, 320 (1987).
[10] T. Kaluza, Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl. **1921**, 966 (1921).
[11] T. Appelquist, A. Chodos, and P.G.O. Freund, *Modern Kaluza-Klein Theories* (Addison-Wesley, Menlo Park, CA, 1987).
[12] R. Geroch, J. Math. Phys. **12**, 918 (1971).
[13] M.E. Fels and C.G. Torre, Class. Quantum Grav. **19**, 641 (2002).
[14] H. He, Y. Ma, and X. Yang, Int. J. Mod. Phys. D (to be published), gr-qc/0212101.
[15] R.M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).